On the Efficiency of Sharing Economy Networks

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Abstract—Exchange of resources (or, services) over sharing economy networks is attracting increasing interest. Despite their broad applicability however, many fundamental questions about their properties remain unanswered. We consider a general sharing economy model and analyze the dynamic interactions of nodes under three different approaches in a stochastic environment. First, we study a centrally designed allocation policy that yields the fair resource each node should receive based on the resources it offers to others. Next, we consider a competitive market where each node determines its allocation strategy so as to maximize the service it receives in return, and a coalitional game model where nodes may coordinate their policies. We prove there is a unique equilibrium exchange allocation for both settings, which also coincides with the central fair allocation. We also characterize the properties of the long-term equilibrium allocations, and analyze their dependency on the network graph. Finally, a dynamic decentralized algorithm is introduced that achieves this desirable operation point with minimal information exchange. The proposed policy is the natural reference point to the various mechanisms that are considered for motivating node collaboration in such networked sharing economy markets.

Index Terms—Network Economics, Sharing Economy, Lexicographic Optimization, Resource Sharing, Virtualization, Coalitional Games, Max-min Fairness, Social Networks, Network Optimization

1 Introduction

1.1 Motivation

Sharing economy is a new economic trend that promotes novel models of sharing, bartering, or renting resources and services, which is opposed to traditional ownership-based models [1]. These solutions have attracted significant interest due to the global recession that has changed the consumer behavior, the pressing environmental concerns, and the penetration of Internet that facilitates such activities [2], [3]. The success of sharing economy is best manifested by the fact that it encompasses very diverse models. In some cases the payments are implemented with legal tender currency while in others the sharing activities are supported by bespoke credit systems; some applications have geographically-restricted scope while others operate in a world-wide scale; and users' collaboration can be decentralized, or mediated by third parties as in transportation apps. These consumption-as-a-service schemes offer sustainable solutions to daunting consumption problems [4].

One of the most promising features of these sharing platforms is that they can leverage the ubiquitous availability of mobile computing and Internet access to support the flexible and direct exchange of resources [5]. Indeed, sharing economy users today have all the means to coordinate and share their resources (or, services) in a dynamic and fully decentralized fashion. The underlying idea is quite simple: whenever a user has some idle resource, she offers it to other users who at that time have unsatisfied needs, expecting to benefit from the resources they in turn will offer to her in the future. Such solutions can improve resource efficiency whenever the users have complementary types of resources, or simply different preferences for them [6].

Not surprisingly, there is already a vast range of decentralized sharing economy models, see, e.g., [1], where users are both resource consumers and producers (or, prosumers), and decide independently how to distribute their spare resources. Moreover, in many cases their collaboration opportunities are constrained by a network graph. For example, ride sharing or food sharing is constrained by the geographic proximity of the participants; renewable energy sharing relies on the grid network; commodity exchanges are conditioned on matching user needs, and so on. We will use hereafter the term *sharing economy networks* to describe these decentralized graph-constrained models where users embedded in a network exchange directly their resources over time, aiming to maximize their individual benefits.

Despite the huge potential of sharing economy networks, important questions about their salient features remain unanswered. For example, it is not understood if these decentralized systems admit *sharing equilibriums* nor if there exist meaningful dynamic sharing policies that can lead to these equilibriums. Also, it is not known how efficient these equilibriums are in terms of social welfare, and whether they are robust to strategic behaviors of users. Finally, a hitherto under-explored question is how the network affects the performance of these systems, and to what extent the network position of a user impacts the resources she receives.

Motivated by these key questions we consider a general model for decentralized sharing economy networks, and we systematically articulate and address the following issues:

(I1)*: Definition and Properties of Fair Sharing.* This is one of the most critical issues in sharing economy. From a system design point of view, each user ideally should receive resource commensurate to its contribution. This is necessary to establish the sense of fairness, yet is not always possible since the network restricts users' collaboration opportunities. There may exist multiple feasible sharing solutions that

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differ on the amount of resource each user receives, and we need to select one that is *fair*, i.e., balances the exchanges as much as possible. In sharing economy networks, such fair allocations are also efficient as they minimize resource waste. The existence and characterization of the properties of centrally designed fair policies (e.g., their dependency on the network graph) is an important open question.

(I2): *Existence and Fairness of Sharing Equilibriums*. Most often these systems are not controlled by a central entity that can impose a fair solution. Instead, each user is free to decide her strategy and therefore allocate her resource to neighbors from which she receives more service. Such interactions give rise to competitive markets where users exchange resources greedily. The main question here is if these myopic strategies lead to an equilibrium allocation where each user cannot unilaterally improve her benefits. In other scenarios, groups of users might be able to form coalitions and exclude non-members from sharing. For example, in a Wi-Fi sharing community such as FON [7] a subset of users may decide to serve only each other, expecting to increase their benefits. Such strategies are likely to deteriorate the system performance and it is important to explore if there are coalitional equilibriums that partition the network. Finally, another key question is how efficient these competitive or coalition equilibriums are, i.e., whether they are related to the above centrally designed fair sharing policy.

(I3): *Dynamics of Sharing Interactions*. In such decentralized systems, the issue of how users can reach the sharing equilibriums is very important from a practical point of view. Namely, we need to understand if there are simple-to-implement dynamic allocation rules which allow users to exchange their randomly created resources in a fair fashion. Such policies must be incentive-compatible, i.e., aligned with each user's efforts to maximize her benefits, and use only minimal information regarding the network structure and the resources or decisions of other users.

The problem of efficient resource sharing has been studied in several contexts. Many communication and computing systems rely on resource pooling, e.g., peer-to-peer file sharing systems [8], [9] and Internet sharing platforms [10], [11]; and similar ideas have been explored for energy sharing in smart grid [12]; cost reduction in services [13]–[15]; and sharing of vehicles [16]. However, these works do not analyze the impact of network graph, nor the users' competitive interactions. Here we study a rich setting, which not only has graph constraints similarly to other networked economies [17], but also does not presume the existence of a monetary instrument. Our focal point is the dynamics of users' decisions, an aspect that remains under-investigated even in general competitive market models [18].

1.2 Methodology and Contributions

In order to shed light on issues ***(I1)-(I3)***, we employ a general model that captures the key properties of *decentralized sharing economy networks*. We consider a set of users, where each one generates over time a random amount of resource that does not consume (excess resource), and therefore can allocate to her neighbors who are interested in it (otherwise it is wasted). We assume that each user has unsaturated demand for the resources of others. Our model captures many practical applications, such as:

- *Time-sharing banks* where people (neighbors; club members; co-employees) exchange their spare time by assisting each other in various tasks; see [19].
- *Energy sharing* in microgrid networks, where residential users take turns in sharing their idle energy with those having unsatisfied needs; see [12].
- *Network sharing*, such as sharing of mobile Internet or Wi-Fi access. There are numerous platforms where users share their connectivity [7], [11]. The design of fair reciprocation mechanisms lies at the core of these systems and remains an open question.
- *Wireless Community Networks*. Users can pool their infrastructure (e.g., routers, antennas, etc.) to build a shared network that can offer, for instance, coverage in rural areas or low-cost Internet access [20]. Deciding how the network capacity should be shared is an inherent problem in the design of such systems.
- *Sharing economy platforms*. Finally, there is a fast-increasing number of online collaboration platforms where users exchange services or commodities, and many of them operate in a fully decentralized fashion; see [1].

In our model the collaboration opportunities are described by an undirected graph. The *sharing ratio* (or, simply *ratio*) of total received over allocated long-term average resource characterizes the performance of each user, as it quantifies the accrued benefits over her contributions. This model can be readily extended for users having different preferences for the different types of resources. The resources can be shared only among one-hop neighbors, and they are directly consumed by their recipients and cannot be distributed farther. This assumption captures practical distribution constraints in sharing systems.

From a system point of view, a network controller would prefer to have a vector of sharing ratios where each coordinate that corresponds to a user has value equal to one. Often this will not be possible due to the graph exchange constraints and asymmetries in nodes' resource availability. For example, in a microgrid energy sharing network, some renewables might create large amounts of energy which cannot be matched by neighboring devices. Hence, the lexicographically maximum (lex-optimal), or max-min, sharing vector is a meaningful performance criterion as it is Pareto optimal and balances the shared resources [21].

In the absence of a controller we assume that each user makes greedy myopic allocations to maximize the aggregate resource she receives in return from others. The interactions of users give rise to a competitive market, which however differ from previous market models [9], [18] due to the existence of the graph, and the absence of side-payments (money) among nodes. We introduce the concept of *sharing equilibrium* that is appropriate for this setting, characterize the resulting equilibrium allocations, and study their relation to the centrally designed max-min fair policy. Our user decision model has been verified by proper behavioral experiments that we have separately executed [22].

Next, we assume that subsets of nodes form coalitions and exchange resources only with each other. A respective coalitional graph-constrained game with non-transferable utility (NTU) is identified. We focus on the existence and properties of stable equilibrium allocations. Given a certain global allocation, if there is a subset of nodes that when they reallocate their own resources among themselves improve the sharing ratio of *at least one* node in the subset, then they have an incentive to deviate from the global allocation. Therefore, when an allocation is in equilibrium it should be *strongly stable* and no such subset should exist.

We study the above frameworks, that differ on the assumptions about the system control and user behavior, and find a surprising connection among them. In particular:

• *(i)* We prove that there is a unique sharing equilibrium ratio vector that is a solution for the competitive market, and lies in the core of the NTU coalitional game, being also strongly stable. This is the max-min fair ratio vector. This result reveals that a centrally designed fair solution is robust to nodes' selfish strategies even if they are allowed to coordinate and form strategic groups seeking to improve their payoff. This finding has many implications for the applicability of such fair policies to sharing economy systems.

• *(ii)* It is shown that the equilibrium exhibits rich structure and a number of interesting properties. For example, in the equilibrium allocation there is exchange of resources only among the nodes with the lowest sharing ratios and the nodes with the highest ratios, the nodes with the second lowest ratios with the set of the second highest ratios, and so on. We also study how the sharing ratios are affected by the graph properties, such as the node degree. This latter aspect is particularly important from a network design point of view as it reveals, among others, the impact link removals or additions have on the equilibrium. Our findings hold for any graph, and therefore they can help a controller to predict or even induce certain equilibriums.

• *(iii)* Finally, we propose a distributed stochastic algorithm that can be used by the nodes to make sharing decisions over time. The algorithms is simple and has minimal information requirements. It allocates the resource generated at each time instance at a node to its neighbor with the highest exchange ratio at that time. This strategy is intuitive since it maximizes the current sharing benefits for each user. Interestingly, it is proved that it also leads to the above fair and robust sharing equilibrium points.

The rest of this paper is organized as follows. In Section 2 we present the dynamic model and problem statement; Section 3 introduces a policy that solves the problem for all three frameworks; Section 4 presents extensive numerical results; and Section 5 surveys the related literature. We conclude in Section 6 where we also discuss our model assumptions. All the proofs can be found in the Appendix.

2 Model and Problem Statement

2.1 Notation and Model

We use capital letters for sequences of random variables, e.g., $\{X(t)\}$, while time averages are denoted with the same letter and a bar, e.g.,

$$\bar{X}(t) = \frac{\sum_{\tau=1}^{t} X(\tau)}{t}.$$

Let $G = (\mathcal{N}, \mathcal{E})$ denote a connected undirected graph with a set $\mathcal{N}$ of N nodes and a set $\mathcal{E} \subseteq \{(i,j) : i,j \in \mathcal{N}, i \neq j\}$ of E links. Each node i represents a user in the considered sharing economy network, and $\mathcal{N}_i$ is the set of its immediate neighbors $\mathcal{N}_i = \{j : (i,j) \in \mathcal{E}\}$. We consider a system that

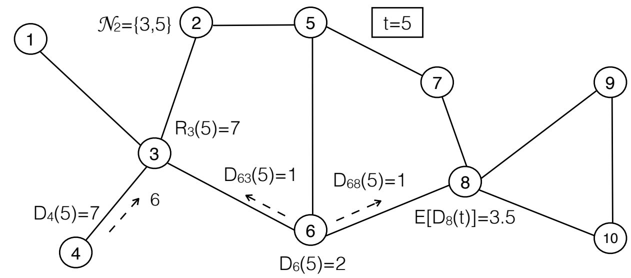

Fig. 1: A sharing economy network example. Dashed arrows show the allocation decisions of nodes at $t = 5$. $D_4(5) = 7$ means that node 4 generated 7 units of excess resource in t; $D_{63}(5) = 1$ states that node 6 allocates 1 resource unit to node 3; and $R_3(5) = 7$ that node 3 receives in total 7 resource units.

evolves over time and we assume a slotted time operation, where slot $t = 1, 2, \cdots$, is the time interval $[t, t+1)$. The "beginning" and "end" of slot t are the times t and $t+1$.

At the beginning of t, node i generates excess resource $D_i(t)$, where $\{D_i(t)\}$ are i.i.d with mean $E[D_i(t)] = D_i > 0$, assumed to be bounded, i.e., there is a real number B such that $D_i(t) \leq B$, $i \in \mathcal{N}$, $\forall t$. The long-term average produced resource by node i, D_i, will be referred to as "endowment" of i. This resource is distributed to its neighbors according to a policy π which is formally defined below.

Definition 1. *A policy π is a set of rules based on which the distribution of resources among the nodes in $\mathcal{N}$ is effected over time. Namely, π determines the amount or resource node i gives to $j \in \mathcal{N}_i$ at time t, based on the generated and allocated resources up to t. We denote the class of all policies by Π.*

Under a policy π, at time t node i gives to j amount $D_{ij}^{\pi}(t) \geq 0$ of the resource it generates at t, and since the node cannot give more than it generates, it holds:

$$\sum_{\tau=1}^{t} \sum_{j \in \mathcal{N}_i} D_{ij}^{\pi}(\tau) \leq \sum_{\tau=1}^{t} D_i(\tau), \quad \forall t. \tag{1}$$

The average resource i gives to its neighbors by t is:

$$\bar{D}_i(t) = \frac{\sum_{\tau=1}^{t} \sum_{j \in \mathcal{N}_i} D_{ij}^{\pi}(\tau)}{t}.$$

The amount of resource node i receives from its neighbors at time t is $R_i^{\pi}(t) = \sum_{j \in \mathcal{N}_i} D_{ji}^{\pi}(t)$ and the average resource received by time t is, $\bar{R}_i^{\pi}(t) = \sum_{\tau=1}^{t} R_i^{\pi}(\tau)/t$. We define the long-term average resource that i receives as $\liminf_{t\to\infty} \bar{R}_i^{\pi}(t) \triangleq r_i^{\pi}$. In general r_i^{π} is a random variable, but in order to obtain policies that satisfy the objectives of interest in this work, it suffices to restrict attention to policies for which $\lim_{t\to\infty} \bar{R}_i^{\pi}(t)$ exists and has a constant and finite value. We provide an example of a network and its operation in slot $t = 5$ in Figure 1.

The set of feasible long-term average received resource vectors that can be achieved by policies in Π is:

$$\mathcal{R} = \{\boldsymbol{r}^{\pi} \triangleq (r_i^{\pi})_{i \in \mathcal{N}} : \pi \in \Pi\}. \tag{2}$$

Node i has unsaturated demand for the resources of its neighbors, and hence it is interested in maximizing its average received resource r_i^{π}. Our model can be directly extended for the case that each node's resource has different significance for the network; or the case each node i obtains different utility for the resource of each other node $j \in \mathcal{N}_i$. We explain in detail this extension in Section 6.

Clearly, the objectives of the nodes are conflicting, since nodes with common neighbors have to compete for resources of the latter. The main question is how one should allocate the resources produced by the nodes. There are two basic approaches to address this issue. First, one could formulate this problem as a centrally defined fair-allocation problem and take into account the resource contribution of each node $i \in \mathcal{N}$ to the community in the long-term, so as to decide how much resource r_i to return to it. From a different perspective, each node is interested in maximizing it own received resource, and this gives rise to competitive interactions and hence creates a sharing economy market. In that case, the amount of resource each node receives depends on the attained equilibrium, if any exists. Additionally, it is possible in some settings for users to coordinate with each other and form sharing groups, or coalitions, aspiring to improve their benefits by excluding non-members. Our goal is to analyze the long-term average performance of the nodes' interactions in all these frameworks.

2.2 Operating Frameworks

2.2.1 Fairness Framework

We consider first a centralized policy that allocates resources to nodes in proportion to their contribution. Ideally, each node i distributes in the long-run all its endowment and it should receive resource commensurate to what it allocates to others, i.e., $r_i = D_i$. However, due to the sharing constraints imposed by the graph, and the different node endowments, such policies might not be feasible. Hence, the designer would prefer to ensure the "most balanced" long-term allocation. A suitable method to achieve this, is to employ the lexicographic optimal criterion, which has been extensively used for load balancing in communication networks [21], [23], [24]. This multi-objective optimization method increases first as much as possible the allocated resource to the node with the smaller sharing ratio, r_i/D_i. Next, if there are many choices, it attempts to increase the resource allocated to the node with the second smaller sharing ratio, and so on. The resulting long-term average allocation is max-min fair, thus as balanced as possible. Next we provide the necessary definitions.

Definition 2. ***Lexicographical order***. *Let $\boldsymbol{x}$ and $\boldsymbol{y}$ be N-dimensional vectors, and $\boldsymbol{\phi}(\boldsymbol{x})$ and $\boldsymbol{\phi}(\boldsymbol{y})$ the N-dimensional vectors that are created by sorting the components of $\boldsymbol{x}$ and $\boldsymbol{y}$, respectively, in non-decreasing order. We say that $\boldsymbol{x}$ is lexicographically larger than $\boldsymbol{y}$, denoted by $\boldsymbol{x} \succ \boldsymbol{y}$, if the first non-zero component of the vector $\phi(\boldsymbol{x}) - \phi(\boldsymbol{y})$ is positive. The notation $\boldsymbol{x} \succeq \boldsymbol{y}$ means that either $\boldsymbol{x} \succ \boldsymbol{y}$, or $\boldsymbol{x} = \boldsymbol{y}$, i.e., $\phi(x) - \phi(y)$ has only zero components.*

Within this framework we are interested to determine policies that induce a lexicographically optimal *sharing ratio* vector. That is, defining $\rho_i^\pi \triangleq \frac{r_i^\pi}{D_i}$, and $\boldsymbol{\rho}^\pi = (\rho_i^\pi)_{i\in\mathcal{N}}$, we are interested in finding a policy π^* such that $\boldsymbol{\rho}^{\pi^*} \succeq \boldsymbol{\rho}^\pi$, for all $\pi \in \Pi$. In the following, a vector $\boldsymbol{r}$ whose sharing ratio vector is lexicographically optimal will also be called simply "lexicographically optimal" or "lex-optimal".

2.2.2 Competitive Framework

Assume now that each node i is an independent decision maker, interested in maximizing r_i. A meaningful decision criterion for i would be to allocate its resources to those nodes $j \in \mathcal{N}_i$ that reciprocate with the higher rate. These are its neighbors which, for every resource unit they receive from i, give back the larger number of units. This effect can be quantified by the exchange or *sharing ratios* of the resources allocated among neighbors since the beginning of time ($t = 1$). Based on these ratios, each node can dynamically distribute its spare resources in order to maximize the anticipated returns. The solution concept for this setup is effectively the competitive equilibrium [18]. However, for the problem under consideration, we introduce a new type of *money-free* equilibrium:

Definition 3. Sharing Equilibrium. *A sharing equilibrium is defined as the vector of sharing ratios $\boldsymbol{\rho}^* = (\rho_i^*)_{i\in\mathcal{N}}$ that satisfy the following properties.*

- (i) *(utility-maximization): node i gives the resources it generates to its neighbors in such a manner that it maximizes its expected received resource r_i;*
- (ii) *(feasibility): the allocation decisions of each node satisfy the constraint that the total allocated resource does not exceed the amount it is entitled by the specified exchange rate and its endowment, that is, $r_i \leq D_i \rho_i^*$;*
- (iii) *(rational behavior): If at time t node i gives resource $D_{ij}(t)$ to node $j \in \mathcal{N}_i$ node i expects in return resource $D_{ij}(t)/\rho_j^*$.*

The above definition is aligned with the competitive equilibrium, and it is easy to see that the respective equilibrium policy π^* will have the following desirable properties:

1) each node distributes all its endowment to its neighbors in the long run, i.e.,
$$\lim_{t\to\infty} \bar{D}_i(t) = D_i, \tag{3}$$
2) each node distributes its resource at all times to the neighbors that have the smallest sharing ratio. Moreover, since each node attempts to maximize its received resource without taking into account the available endowments of its neighbors, the optimization should result in received resource rate vector $\boldsymbol{r}^*$ that satisfies $r_i^* = D_i \rho_i^*,\ i \in \mathcal{N}$.

In this context, we are interested in determining whether equilibrium sharing ratios and associated policies exist. Moreover, we are interested in dynamic and decentralized policies that operate without a priori knowledge of the equilibrium rates (provided that they exist), but adjust the sharing ratios over time in such a manner that they eventually converge to the equilibrium ones.

2.2.3 Coalitional Framework

Let $G_\mathcal{S} = (\mathcal{S}, \mathcal{E}_\mathcal{S})$ denote the subgraph of G induced by a nonempty set of nodes $\mathcal{S} \subseteq \mathcal{N}$, i.e., the graph with nodes $\mathcal{S}$ and links $\mathcal{E}_\mathcal{S} = \{(i,j) : i, j \in \mathcal{S}\}$. $\Pi_\mathcal{S}$ denotes the set of policies that operate on graph $G_\mathcal{S}$, and $\mathcal{R}_\mathcal{S}$ is the set of all long-term received resource vectors such that:
$$\mathcal{R}_\mathcal{S} = \{\boldsymbol{r}^\pi \triangleq (r_i^\pi)_{i\in S} : \ \pi \in \Pi_\mathcal{S}\}.$$

Note that $G_{\mathcal{S}}$ may not be connected. However, the definition of policy in Sec. 2.1 still holds, and hence the set $\Pi_{\mathcal{S}}$ is well defined. Also, all the stated results for connected graphs hold for each of the connected components of $G_{\mathcal{S}}$.

In this setting we assume that subsets of nodes can form coalitions and deviate from the fair solution if this will ensure higher resources for some of them. In game theoretic terms, this behavior leads to a coalitional game [53] played by the nodes. Specifically, we call any nonempty subset of nodes $\mathcal{S} \subseteq \mathcal{N}$ a coalition when they allocate their resources only among each other. That is, there is no resource exchange among nodes in $\mathcal{S}$ and those in set $\mathcal{S}^c = \mathcal{N} \setminus \mathcal{S}$. Hence, the feasible long-term resource vectors that nodes in $\mathcal{S}$ get are the $|\mathcal{S}|-$dimensional vectors in $\mathcal{R}_{\mathcal{S}}$. We refer to set $\mathcal{N}$ as the *grand coalition*. This coalitional game is one with non-transferable utilities, as resources cannot be split among the nodes due to the network exchange constraints. Our goal is to study the existence and properties of self-enforcing long-term allocations. This property is captured by the notion of stability for the grand coalition.

Definition 4. ***Coalitional Stability***. *A grand coalition $\mathcal{N}$ with a policy $\pi^* \in \Pi$ that induces long-term received resource vector $\boldsymbol{r}^*$ is called* strongly *stable if for any nonempty node set $\mathcal{S} \subseteq \mathcal{N}$, there is no policy $\pi_{\mathcal{S}} \in \Pi_{\mathcal{S}}$ that induces an $|\mathcal{S}|-$dimensional vector $\boldsymbol{r}$ such that $r_i \geq r_i^*$ for all $i \in \mathcal{S}$, and $r_j > r_j^*$ for at least one node $j \in \mathcal{S}$. The allocation is called* weakly *stable if for any nonempty node set $\mathcal{S} \subseteq \mathcal{N}$, there is no policy $\pi_{\mathcal{S}} \in \Pi_{\mathcal{S}}$ that induces a vector $\boldsymbol{r}^{\mathcal{S}}$ such that $r_i^{\mathcal{S}} > r_i^*$ for all $i \in \mathcal{S}$.*

Note that strong stability implies weak stability but not the other way around. In particular, the concept of weak stability for the grand coalition is directly related to the *core* concept. In this framework we ask the question: is there a policy $\pi \in \Pi$ that renders the grand coalition stable?

3 A Unifying Policy For The Frameworks

3.1 Main Result

We describe a simple policy π^* that achieves the objectives of these three frameworks. According to π^* each node maintains a ratio $\rho_i(t) = \bar{R}_i(t)/D_i$ which may be interpreted as resource sharing ratio (or simply "ratio") at time t. Every node gives its resource generated at t to the node that has the smallest ratio among its neighbors at that point; see Algorithm 1. The only information required for this policy is the node endowments. However, as we will see in Section 6, the policy can also operate by replacing D_i with the running average $\bar{D}_i(t) = \sum_{\tau=1}^{t} D_i(\tau)/t$. The next theorem is the main result of this work.

Theorem 1. *Equivalence of solutions:*

- *Policy π^* is Lexicographically optimal.*
- *Under π^* the node sharing ratios and long-term received resources converge to the equilibrium sharing ratios and equilibrium received resources.*
- *Policy π^* is coalitionally stable.*

Next we present an outline of the arguments that will be used to prove this theorem. In Section 3.2 we show that the region $\mathcal{R}$ in (2) is a subset of a polymatroid, and this allows us to restrict attention to policies under which all nodes distribute their endowments to their neighbors in the long run. Using the structure of the lex-optimal vector in polymatroids, we analyze in Section 3.4 the properties of the optimal point, and we use them to show that a lex-optimal policy achieves also the equilibrium sharing ratios. Finally, in the Appendix we show that policy π^* achieves the lexicographically optimal point and thus possesses all properties described in Theorem 1.

Algorithm 1: Algorithm for implementing policy π^*

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At time t = 1 set ρ_i(t) = 0, i ∈ N.
for t = 1, 2, ... do
    Each user i ∈ N announces to its neighbors the
    sharing ratio ρ_i(t) = R̄_i(t)/D_i;
    Each user i ∈ N distributes the resource it
    generates at time t to its neighbor(s) j ∈ N_i having
    the smallest sharing ratio ρ_j(t) in set N_i.
end
```

3.2 Achievable Received Resource Vectors

For a set $\mathcal{S} \subseteq \mathcal{N}$ define by $\mathcal{N}_{\mathcal{S}}$ the set of nodes that are neighbors of nodes in $\mathcal{S}$, i.e., $\mathcal{N}_{\mathcal{S}} = \cup_{i \in \mathcal{S}} \mathcal{N}_i, \ \mathcal{N}_{\emptyset} = \emptyset$. Let,

$$f(\mathcal{S}) = \sum_{i \in \mathcal{N}_{\mathcal{S}}} D_i,$$

where for $\mathcal{G} = \emptyset$ we define $\sum_{i \in \mathcal{G}} x_i = 0$. Since there are no isolated nodes, it is $f(\mathcal{N}) = \sum_{i \in \mathcal{N}} D_i$. The following lemma stems from the fact that nodes can receive resources only from their neighbors.

Lemma 1. *Under any policy $\pi \in \Pi$ it holds for any $\mathcal{S} \subseteq \mathcal{N}$,*

$$\limsup_{t \to \infty} \sum_{i \in \mathcal{S}} \bar{R}_i^{\pi}(t) \leq f(\mathcal{S}), \tag{4}$$

$$\sum_{i \in \mathcal{S}} r_i^{\pi} \leq f(\mathcal{S}). \tag{5}$$

Let Π_0 be the class of policies in Π for which

1) The long-term average of received resources exist, i.e, $r_i^{\pi} = \lim_{t \to \infty} \sum_{\tau=1}^{t} R_i^{\pi}(\tau)/t, \ i \in \mathcal{N}$.
2) All endowments generated by the nodes are eventually consumed, i.e., $\sum_{i \in \mathcal{N}} r_i^{\pi} = f(\mathcal{N})$.

Let $\mathcal{R}_0$ be the set of received resource vectors that can be achieved by policies in Π_0. From Lemma 1 we conclude:

$$\mathcal{R} \subseteq \left\{ \boldsymbol{r} \geq \boldsymbol{0} : \sum_{i \in \mathcal{S}} r_i \leq f(\mathcal{S}), \ \mathcal{S} \subseteq \mathcal{N} \right\} \triangleq \mathcal{A}, \tag{6}$$

$$\mathcal{R}_0 \subseteq \left\{ \boldsymbol{r} \geq \boldsymbol{0} : \sum_{i \in \mathcal{S}} r_i \leq f(\mathcal{S}), \ \mathcal{S} \subset \mathcal{N}, \sum_{i \in \mathcal{N}} r_i = f(\mathcal{N}) \right\} \triangleq \mathcal{A}_0. \tag{7}$$

To proceed, we need to show that $f(\mathcal{S})$ is submodular.

Lemma 2. *$f(\mathcal{S})$ is submodular i.e., it holds for every $\mathcal{S}, \ \mathcal{T} \subseteq \mathcal{N}$,*

$$f(\mathcal{S} \cap \mathcal{T}) + f(\mathcal{S} \cup \mathcal{T}) \leq f(\mathcal{S}) + f(\mathcal{T}). \tag{8}$$

For submodular $f(\mathcal{S})$, the sets $\mathcal{A}$ and $\mathcal{A}_0$ are referred to as "polymatroid polyhedron" and "base of the polymatroid" respectively. Using the polymatroid property, the next lemma shows that the achievable resource vectors under policies in Π_0 are the base of the polymatroid.

Lemma 3. *It holds:* $\mathcal{R}_0 = \mathcal{A}_0$.

3.3 Review of Polymatroid Properties

We next present some key properties of polymatroids that are needed for our analysis; see also [25], [26].

Lemma 4. *If* $\mathcal{A}$ *is a polymatoid with base* $\mathcal{A}_0$*, then for any* $\boldsymbol{r} \in \mathcal{A}$ *there exist an* $\boldsymbol{r}_0 \in \mathcal{A}_0$ *such that* $\boldsymbol{r}_0 \leq \boldsymbol{r}$*. Hence the lexicographically optimal vector in* $\mathcal{A}$ *lies in* $\mathcal{A}_0$*.*

Next we describe the structure of the lexicographically optimal vector in $\mathcal{A}$. First we need additional notation. For a given $\boldsymbol{r} \in \mathcal{A}$, the *different values* the coordinates of vector $\boldsymbol{\rho} = (\rho_i)_{i\in\mathcal{N}} = (r_i/D_i)_{i\in\mathcal{N}}$ take will be denoted by $v_k(\boldsymbol{r})$, $i = 1,...K(\boldsymbol{r}) \leq N$, where $v_1(\boldsymbol{r}) < v_2(\boldsymbol{r}) < ... < v_{K(\boldsymbol{r})}(\boldsymbol{r})$. The index of the value to which ρ_i equals is denoted by $I_i(\boldsymbol{r})$, i.e., $v_{I_i(\boldsymbol{r})} = \rho_i$. We call $I_i(\boldsymbol{r})$ the "level of node i". The set of nodes of level k is denoted by $\mathcal{L}_k(\boldsymbol{r}) = \{i \in \mathcal{N} : \ I_i(\boldsymbol{r}) = k\}$.

Theorem 2. *Let* $\mathcal{A}$ *be a polymatroid. A vector* $\boldsymbol{r}$ *in* $\mathcal{A}$*, is lexicographically optimal if and only if the following hold.*

$$\sum_{i\in\mathcal{L}_1} r_i = f(\mathcal{L}_1), \tag{9}$$

$$\sum_{i\in\mathcal{L}_k} r_i = f\left(\cup_{l=1}^{k}\mathcal{L}_l\right) - f\left(\cup_{l=1}^{k-1}\mathcal{L}_l\right), \ 2 \leq k \leq K, \tag{10}$$

where, $\mathcal{L}_k = \mathcal{L}_k(\boldsymbol{r})$ *.* $K = K(\boldsymbol{r})$ *. The lexicographically optimal vector exists and is unique.*

3.4 Structure of Lex-Optimal Resource Vector

In this section we describe the structure of the lex-optimal vector. We will use the following simple lemma.

Lemma 5. *Let* $\boldsymbol{r} \in \mathcal{A}_0$ *then: a) If* $K = 1$ *then* $v_1 = 1$*. b) If* $K > 1$ *, then* $v_1 < 1$ *and* $l_K > 1$*.*

For the problem under consideration it can be seen that given any vector $\boldsymbol{r}$ in $\mathcal{A}_0$ there is an allocation set $\{d_{ij} \geq 0, \ i \in \mathcal{N}, \ j \in \mathcal{N}_i\}$ such that $\sum_{j\in\mathcal{N}_i} d_{ij} = D_i, \ i \in \mathcal{N}$ and $\sum_{j\in\mathcal{N}_i} d_{ji} = r_i, \ i \in \mathcal{N}$; we refer to this set as "allocation that generates $\boldsymbol{r}$". This allocation may not be unique, e.g., see Figure 2. Fixing any such allocation, we say that (under this allocation) "node i gives resource to node j" if $d_{ij} > 0$. We also say that "node i gives resource to a set $\mathcal{S}$", if node i gives resource to any node in $\mathcal{S}$.

Consider now the general structure of the lexicographically optimal vector of Theorem 2. Equality (9) implies that there is (at least one) allocation set $\{d_{ij} \geq 0, \ i \in \mathcal{N}, \ j \in \mathcal{N}_i\}$, such that the endowments of all neighbors of set $\mathcal{L}_1$ are given to the nodes in this set. Similarly, for $k = 2$, (10) implies that the endowments of all nodes in $\mathcal{N}_{\mathcal{L}_2} - \mathcal{N}_{\mathcal{L}_1}$ are given to nodes in set $\mathcal{L}_2$. In general, the endowments of all nodes in $\mathcal{N}_{\mathcal{L}_k} - \mathcal{N}_{\cup_{l=1}^{k-1}\mathcal{L}_l}$ are given to nodes in set $\mathcal{L}_k$. Given a vector $\boldsymbol{r}$ with $K(\boldsymbol{r}) \geq 2$, let us define for every $k \in [1, \lceil K/2 \rceil]$:

$$\mathcal{Q}_k(\boldsymbol{r}) = \mathcal{N} - \cup_{m=1}^{k-1}\left(\mathcal{L}_m(\boldsymbol{r}) \cup \mathcal{L}_{K-m+1}(\boldsymbol{r})\right), \tag{11}$$

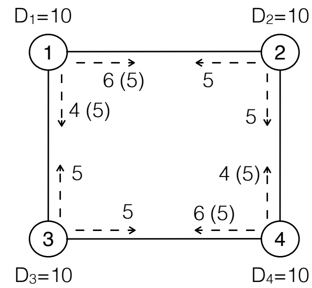


Fig. 2: Example of Multiple Allocations leading to identical sharing ratios. Numbers show main allocation rates, and alternative options shown in parentheses. All nodes have the same average resource generation rate. Both allocations lead to the same exchage ratio vector of (1,1,1,1).

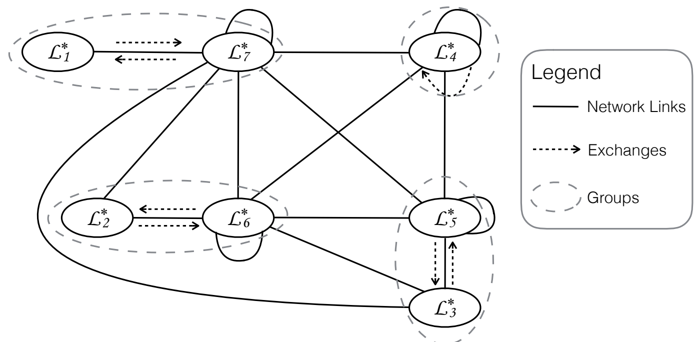


Fig. 3: Structure of a graph with 7 levels. Solid lines show the possible exchanges (physical connections), while dashed arrows the actual exchanges. Note that not all connections are used, as some players do not interact with each other.

where $\cup_{m=k}^{l}\mathcal{S} = \emptyset$ if $l < k$. For example, $\mathcal{Q}_1(\boldsymbol{r}) = \mathcal{N}$, and $\mathcal{Q}_2(\boldsymbol{r})$ consists of the nodes in $\mathcal{N}$ that remain after removing those that belong to level sets $\mathcal{L}_1(\boldsymbol{r})$ and $\mathcal{L}_K(\boldsymbol{r})$. In the sequel, a quantity X referring to induced subgraph $G_{\mathcal{Q}_k(\boldsymbol{r})} = (\mathcal{Q}_k(\boldsymbol{r}), \mathcal{E}_{\mathcal{Q}_k}(\boldsymbol{r}))$ is denoted $X_{\mathcal{Q}_k(\boldsymbol{r})}$. Henceforth, we will omit from notation the dependence on vector $\boldsymbol{r}$.

The next Theorem describes the structure of the lexicographically optimal vector.

Theorem 3. *A vector* $\boldsymbol{r} \in \mathcal{A}$*, is lexicographically optimal if and only if the following hold. If* $K = 1$*, then* $v_1 = 1$*. If* $K \geq 2$ *then*

1) $\mathcal{L}_k$ *is an independent set in graph* $G_{\mathcal{Q}_k}$*, for* $k = 1, ..., \lfloor \frac{K}{2} \rfloor$.
2) $\mathcal{L}_{K-k+1} = \mathcal{N}_{\mathcal{Q}_k}(\mathcal{L}_k)$*, for* $k = 1, ..., \lfloor \frac{K}{2} \rfloor$.
3) $v_k v_{K-k+1} = 1$*, for* $k = 1, ..., \lfloor K/2 \rfloor$.
4) $\sum_{i\in\mathcal{L}_k} r_i = \sum_{i\in\mathcal{L}_{K-k+1}} D_i$*, for* $k = 1, ..., \lfloor \frac{K}{2} \rfloor$.
5) *If* K *is odd, then* $v_{\lceil K/2 \rceil} = 1$.

Let us now discuss the implications of this theorem. Under a lexicographically optimal allocation $\boldsymbol{r}^*$, the nodes are divided in disjoint sets $\mathcal{L}_1^*, \cdots, \mathcal{L}_{K^*}^*$, where nodes in each set have the same sharing ratio. This division depends both on the node endowments and on graph G. For the discussion below, please refer to Fig. 3 that presents an example of the structure for $K^* = 7$ levels. In this graph we abstract with solid lines the physical connections that may exist among the different sets of nodes. The actual nodes and their connections are not shown.

Sharing ratios Structure. According to Theorem 3, the highest sharing ratio is inversely proportional to the lowest one ($l_7^* = 1/l_1^*$), the second highest ratio is inversely proportional to the second lowest ratio ($l_6^* = 1/l_2^*$), and so on. Additionally, as shown in Fig. 3 all nodes in the set with the highest ratio $\mathcal{L}_7^*$, exchange resources only with nodes belonging to the set with the lowest ratio $\mathcal{L}_1^*$. Similarly, nodes in set $\mathcal{L}_6^*$ exchange resources only with nodes in $\mathcal{L}_2^*$, and so on. When K^* is odd, there is one set of nodes, here $\mathcal{L}_4^*$, which exchange resources only with each other.

Topological Properties. Nodes in the set with the lowest exchange rate, $\mathcal{L}_1^*$, constitute an independent set. Moreover, their neighbors are those with the highest sharing ratio, $\mathcal{L}_{K^*}^*$. Similarly, it is $\mathcal{L}_6^* = \mathcal{N}_{\mathcal{Q}_2}(\mathcal{L}_2^*)$ and nodes in $\mathcal{L}_2^*$ constitute an independent set in graph $G_{\mathcal{Q}_2(\boldsymbol{r}^*)}$. Hence, the nodes in set $\mathcal{L}_2^*$ can have links only with nodes in $\mathcal{L}_6^*$ and possibly with nodes in $\mathcal{L}_7^*$ (since the latter do not belong in $G_{\mathcal{Q}_2(\boldsymbol{r}^*)}$). However, as discussed above, nodes in $\mathcal{L}_2^*$ exchange resource only with nodes in $\mathcal{L}_6^*$. With the same reasoning, it is easy to see that nodes in $\mathcal{L}_3^*$ can be physically connected with nodes in $\mathcal{L}_7^*$, $\mathcal{L}_6^*$ and $\mathcal{L}_5^*$, but they exchange resources only with nodes in $\mathcal{L}_5^*$.

These properties reveal how the graph affects the lex-optimal fair solution. For example, by adding a link between two nodes initially belonging to $\mathcal{L}_1^*$, the lex-optimal solution changes and places these (now connected) nodes to another set. This relation between the graph and the lex-optimal sharing vector will become more evident below.

The next theorem shows that the ratios of the lexicographically optimal point are the equilibrium sharing ratios of the competitive framework.

Theorem 4. *Let $\boldsymbol{r}^*$ be a lexicographically optimal vector. The ratios $\{\rho_i^*\}_{i\in\mathcal{N}} = \{r_i^*/D_i\}_{i\in\mathcal{N}}$ are equilibrium sharing ratios for the competitive framework.*

The next theorem shows that a policy achieving the lexicographically optimal vector $\boldsymbol{r}^*$ is stable.

Theorem 5. *A policy π^* that achieves the lexicographically optimal vector $\boldsymbol{r}^*$is strongly stable.*

Next section presents a detailed numerical investigation using small-scale networks with stylized structure (e.g., star topology), and networks created by standard models such as Erdos-Renyi, Scale-free and Small-world graphs.

4 Numerical Results and Discussion

We present a battery of numerical tests using basic network graphs and also standard network generation models.

4.1 Basic Graphs

Consider first the networks of Figure 4. Solid lines represent the physical connections among the nodes, i.e., the possible exchanges, and the dotted arrows indicate the equilibrium resource allocations. Next to each node we depict its resource endowment.

Focus on the 6-node network of Figure 4(a). At the lex-optimal equilibrium, this network has $K^* = 3$ levels with sets $\mathcal{L}_1^* = \{1, 6\}$, $\mathcal{L}_2^* = \{3, 4\}$, $\mathcal{L}_3^* = \{2, 5\}$ which are marked with different colors, where darker colors are used for nodes with higher sharing ratios. We now verify the properties of the lex-optimal allocation, based on Theorem 3. First, notice that $\mathcal{L}_1^*$ is an independent set in graph G. Moreover, the neighbors of nodes in $\mathcal{L}_1^*$ are the nodes in $\mathcal{L}_3^*$. Although nodes in $\mathcal{L}_3^*$ are connected to each other, they only allocate resource to nodes in $\mathcal{L}_1^*$ and it holds $\sum_{i\in\mathcal{L}_3^*} D_i = \sum_{i\in l_1} r_i = 20 + 30$. Moreover, the highest and lowest levels satisfy the condition $u_1^* u_3^* = 1$. The nodes are partitioned into 2 disjoint groups $\mathcal{M}_1^* = \mathcal{L}_1^* \cup \mathcal{L}_3^*$ and $\mathcal{M}_2^* = \mathcal{L}_2^*$, each one containing nodes with at most two levels.

For the example of Figure 4(b) we used a 13-nodes network that yields $K^* = 6$ levels, with $u_1^* = 0.25$, $u_2^* = 0.43$, $u_3^* = 0.77$, $u_4^* = 2.34$, $u_5^* = 1.3$, $u_6^* = 4$. The sets are $\mathcal{L}_1^* = \{12, 13\}$, $\mathcal{L}_2^* = \{4, 6, 8, 10\}$, $\mathcal{L}_3^* = 2$, $\mathcal{L}_4^* = 1$, $\mathcal{L}_5^* = \{3, 5, 7, 9\}$, and $\mathcal{L}_6^* = 11$. Sets $\mathcal{L}_1^*$, $\mathcal{L}_2^*$, and $\mathcal{L}_3^*$ are independent in graphs $G_{\mathcal{Q}_1}$, $G_{\mathcal{Q}_2}$, and $G_{\mathcal{Q}_3}$ respectively, and the set $\mathcal{L}_1^* \cup \mathcal{L}_2^* \cup \mathcal{L}_3^*$ is independent in G. Moreover, it holds $\mathcal{L}_6^* = \mathcal{N}_{\mathcal{Q}_1}(\mathcal{L}_1^*)$, $\mathcal{L}_5^* = \mathcal{N}_{\mathcal{Q}_2}(\mathcal{L}_2^*)$ and $\mathcal{L}_4^* = \mathcal{N}_{\mathcal{Q}_3}(\mathcal{L}_3^*)$, and also $u_6^* u_1^* = u_5^* u_2^* = u_4^* u_3^* = 1$. In this example we have 3 disjoint groups $\mathcal{M}_1^* = \mathcal{L}_1^* \cup \mathcal{L}_6^*$, $\mathcal{M}_2^* = \mathcal{L}_2^* \cup \mathcal{L}_5^*$, and $\mathcal{M}_3^* = \mathcal{L}_3^* \cup \mathcal{L}_4^*$. We see that links $(10, 11)$, $(5, 11)$, $(1, 3)$, $(1, 5)$ and $(2, 7)$ are redundant and can be removed without affecting the equilibrium allocation.

Figure 4(c) depicts a complete graph with 6 nodes, where node $i = 4$ has level $u_{I_1(r^*)} = 0.988$ while the other nodes have level $u_{I_2(r^*)} = 1.012$. In general for complete graphs, from Theorem 3 and the fact that independent sets in such graphs contain only one node, it follows that lex-optimal allocations may have at most two levels. Moreover a complete graph has two levels iff the resource of node i_0 with the maximum endowment is larger than the sum of the resources of the rest of nodes, and it is $\mathcal{L}_1 = \{i_0\}$. On the other hand, for the respective 6-node ring graph, Figure 4(d), the lex-optimal solution yields 4 levels.

4.2 Typical Network Models

We now focus on larger graphs of typical models, namely the Lattice, Erdos-Renyi [27], Scale-free [28], and Small-world [29] networks. We demonstrate that the reached equilibrium points can be affected by the properties of these networks, e.g., their density, but also by the diversity in the nodes' resource endowments. In Figure 5 we present the equilibrium allocations for 3 lattice graphs with 30 nodes. Figure 5a presents the homogeneous case where every node has endowment equal to 30. We observe that the graph structure does not create any imbalance in the equilibrium and all nodes achieve sharing ratio 1. This result changes significantly when the nodes have diverse endowments. Namely, in Figure 5b we depict the equilibrium of the same lattice network where 2 out of the 30 nodes have now higher endowment of 300 units. This creates 7 different levels. Finally, Figure 5c depicts a graph where 5 out of the 30 nodes have resource $D_i = 300$, and this creates 4 different sharing ratio levels, making the sharing economy network less imbalanced in that respect.

Next we focus on Erdos-Renyi (E-R) graphs. Figure 6a depicts an E-R graph with 30 nodes with equal endowment, $D = 30$. We observe that all nodes reach the same unique equilibrium point of equal exchanges. The same holds for Figure 6b which is denser ($p = 0.2$), yet the additional links do not affect the nodes' allocation strategies. However, when the nodes become resource-diverse with 2 nodes

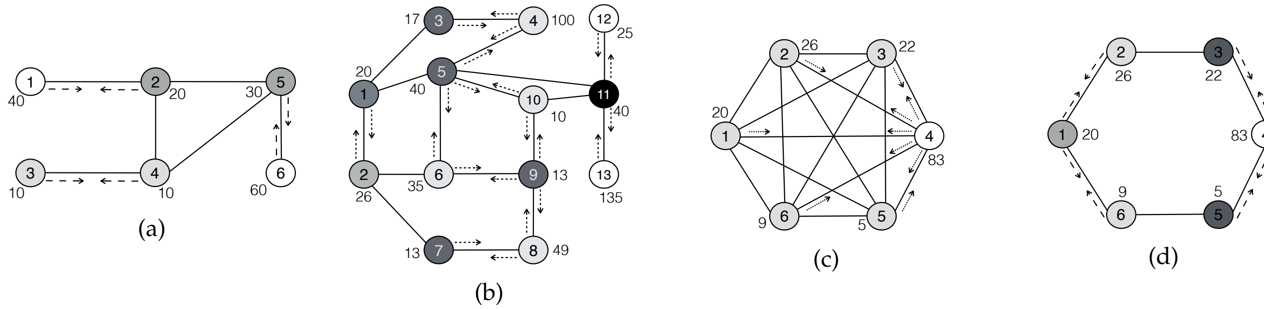


Fig. 4: **(a)**: A network with 6 nodes that create 2 groups. There are 3 different levels with sharing ratios $u_1^* = 0.5$, $u_2^* = 1$, and $u_3^* = 2$. The color of each node is analogous to its sharing ratio value (increasing from white to black). The received resources are $(r_1^*, r_2^*, r_3^*, r_4^*, r_5^*, r_6^*) = (20, 40, 10, 10, 60, 30)$; **(b)**: A network with 13 nodes which create 3 groups. Received resources are $(r_1^* : r_{13}^*) = (26, 20, 39.7, 42.8, 93.5, 15, 30.4, 21, 30.4, 4.3, 160, 6.3, 33.8)$. **(c)**: A complete graph of 6 nodes with 1 coalition and 2 levels. **(d)**: A ring graph of 6 nodes with 2 coalitions and 4 levels.

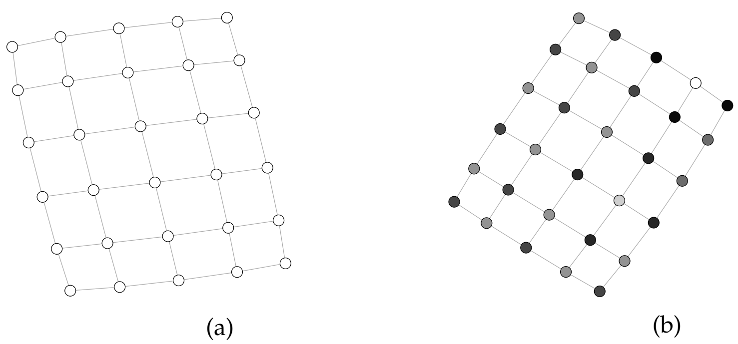


Fig. 5: Lattice with 30 nodes. (a): Equally endowed nodes ($D = 30$), 1 level. (b): 28 low-endowed ($D = 30$) and 2 high-endowed nodes ($D=300$), 7 levels. (c): 25 low-endowed (30) and 5 high-endowed nodes ($D=300$), 4 levels.

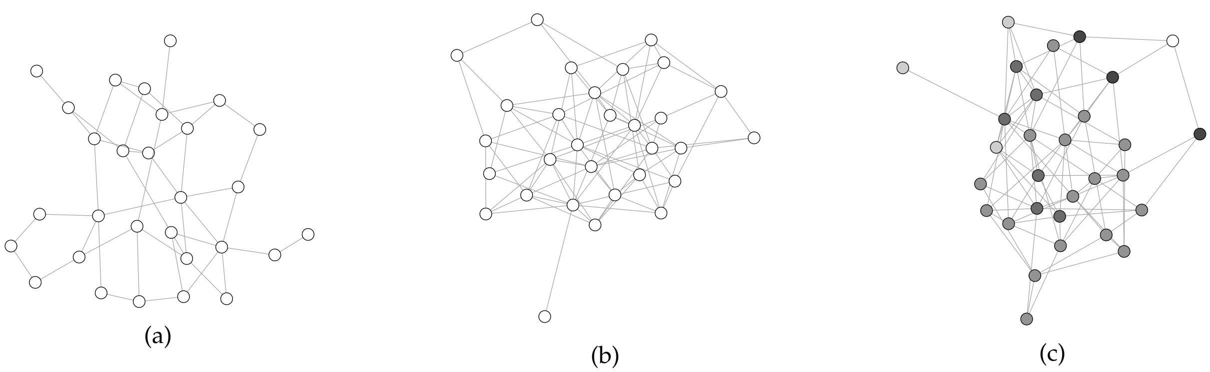
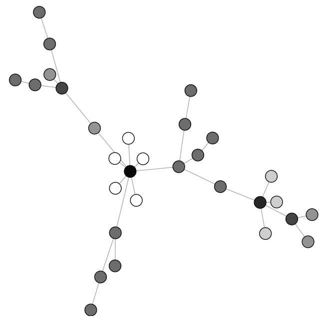


Fig. 6: Equilibriums in Erdos-Renyi Graphs. (a): $N = 30$, link creation probability $p = 0.1$, 1 sharing ratio level. (b): $N = 30$, $p = 0.2$, 1 sharing ratio level. (c): $p = 0.2$, 2 high-endowment nodes ($D = 300$), 4 sharing ratio levels.

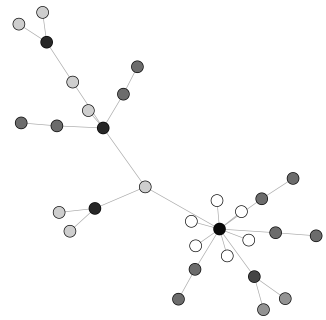
having $D_i = 300$, Figure 6c, there are 5 levels ranging from $u_1 = 0.33$ up to $u_5 = 3.33$. In other words, similarly to the lattice graph we observe that a change in the resource endowments is more likely to change the equilibrium than a change in the structure of these examples.

This is not the case however for scale-free graphs which are formed through a preferential attachment process. These networks not exhibit degree assortative mixing [54], since many nodes with low degree are connected to nodes with high degree. This creates a structural advantage which results in diverse equilibrium sharing ratios even when nodes have identical endowments. Figure 7a presents the equilibrium in a scale-free graph with power parameter $k=0.5$, yielding 5 levels. Figure 7b presents a graph created by a linear preferential attachment process, i.e., $k = 1$, that attains an equilibrium with 7 levels. Finally, for the graph of Figure 7c it is $k = 2$ and the equilibrium yields 5 levels. In summary, we see with this basic example that in scale-free graphs the equilibrium is significantly affected by the structural properties of the network graph and result in asymmetric points even when the nodes are identical in terms of their resource endowments. We have also observed this result in our pertinent behavioral experiments involving human subjects making Wi-Fi sharing decisions [22].

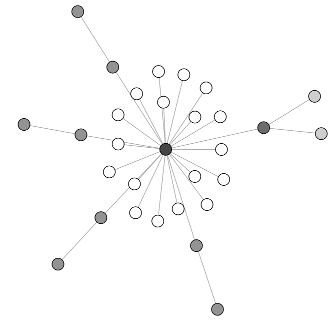
As a final example, we present in Figure 8 the equilibriums in 30-node networks that have the small-world prop-



(a)

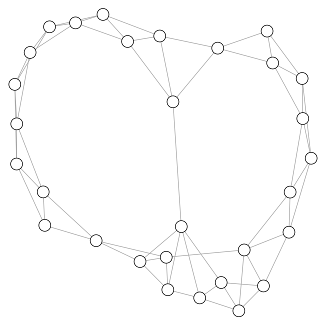


(b)

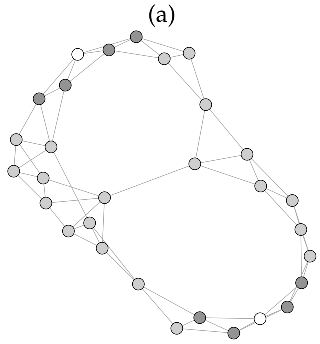


(c)

Fig. 7: Scale-free Graphs. (a): low skewed with skew parameter 0.5, 5 sharing ratio levels. (b): moderately skewed (linear model, skew parameter 1), 7 sharing ratio levels. (c): highly skewed (parameter 2), 5 sharing ratio levels.



(a)



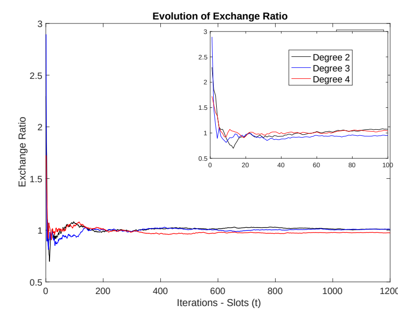
(b)

Fig. 8: Small-world Graph, N=30. (a): Small-world Graph with symmetric nodes; (b): Small-world Graph with Asymmetric Endowments (4 nodes with D=300)

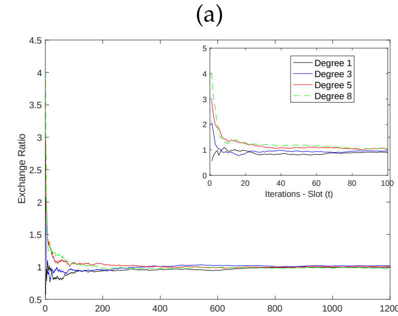
erty [29]. Figure 8a presents a homogeneous network where all nodes have equal resource endowments (30 units) while Figure 8b depicts the same network where 4 nodes have endowment equal to 300. This creates 3 different sharing ratio levels, instead of a single level for the former case.

4.3 Dynamic Interactions and Convergence

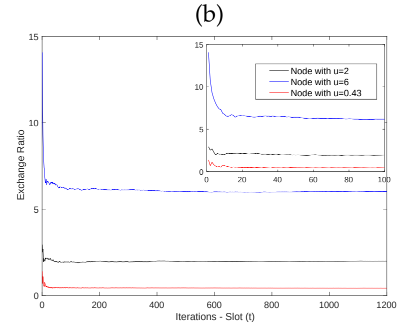
We present the convergence results for Algorithm 1. Figure 9a presents the value over time of the sharing ratio $\rho_i(t) = \bar{R}_i(t)/D_i$ for three nodes in the lattice network of Figure 5a. We observe that after 100 slots (see the inset) the ratios have converged very closely to their final values. We have plotted the results for nodes with different degrees, which however does affect in this example the convergence speed. Similarly, in Figure 9b we present the convergence of



(a)



(b)



(c)

Fig. 9: Convergence of Algorithm 1. (a):Convergence of sharing ratios in the Lattice of Fig. 5a; results shown for 3 nodes with degrees 2, 3, and 4. Inset plot shows the first 100 iterations. (b): Convergence of sharing ratios in the Erdos-Renyi Network of Fig. 6a; results shown for 4 nodes with degrees 1, 3, 5 and 8. Inset plot shows the first 100 iterations. (c): Convergence of sharing ratios in the Scale-free Network of Fig. 7b; results shown for 3 nodes with 3 different sharing ratios. Inset plot shows the first 100 iterations.

4 nodes with different degrees in the Erdos-Renyi network of Figure 6a. Again we observe a relatively fast convergence on the final sharing ratios which are all equal to 1. Finally, Figure 9c presents the convergence results for 3 nodes in the Scale-free graph of the network in Figure 7b. The sharing ratios converge in different levels, and we show here the results for 3 nodes attaining 3 different sharing ratio values.

5 Literature Overview

The questions considered in this work are fundamental and arise in different contexts. Similar cooperation models have been studied in network exchange theory in sociology; the general equilibrium theory in economics; game theoretic analyses of networked markets; and in the design of resource sharing mechanisms for communication and computing systems. One novel and distinguishing aspect of our model is that node resources are initially unknown and revealed only in each slot. This make our results applicable to a very broad set of applications.

5.1 General Equilibrium and Game Theory Models

Our work is directly related to the general equilibrium theory that focuses on the existence of allocation equilibriums in competitive markets. The first attempt to study such models dates back to 1874 and the efforts of L. Walras [30] who introduced the tatonnement process that leads to an equilibrium; and the analysis of I. Fisher who considered a simpler model in 1891 [31] and studied automated methods for computing the equilibriums. A richer macroeconomic model was introduced in the seminal paper of K. Arrow and G. Debreu [18] who formally proved the existence of equilibriums (A-D model). Subsequent works refined these results by devising the necessary and sufficient conditions for the existence of equilibriums when the agents have linear utility functions [32]. More recently, researchers have focused on the computation complexity of A-D and Fisher equilibriums, developing approximate or exact (pseudo)-polynomial algorithms for special cases [33]–[35]. We refer the reader to [36] for an excellent discussion and overview of recent literature.

The sharing economy model in this paper differs from the above works in many ways. First, we do not assume the existence of any monetary instrument; hence commodity prices cannot be introduced nor we need to calculate them in order to find the economy's equilibriums. In other words, this is a pure exchange or bartering scheme. Similar models have been studied for house exchange markets [37] or labor (timeshare) exchanges [19]. Another distinct aspect of our model is the graph that constraints the possible trades. It is worthwhile to note here that A-D and Fisher markets can capture to some extent the network constraints through the commodity preferences if we assume that each agent possesses a different commodity. However, even under this assumption, previous works did not study the impact of the preference constraints on the equilibriums. Similar network models have been studied in the context of graphical economies [17], which revealed that the graph can create local variations in the price of each commodity.

Compared to these latter results in our model there is no budget constraints (as in typical exchange economies) and the nodes do not accrue utility from the money per se. We also prove that the sharing equilibriums lie within the core of the respective NTU game. Although this relation is known for market and coalitional games, to the best of our knowledge this is the first result for NTU coalitional graph-constrained games without money transfers. Moreover, we focus on market clearing (or competitive) equilibriums which rely on the assumption of price-taking behavior for the agents, hence are fundamentally different from Nash equilibriums. Finally, [38] and [39] studied core solutions of coalitional graph games where the nodes are allowed to create new or severe existing connections. In our model the graph is exogenously given, e.g., based on the location of nodes.

5.2 Dynamic Sharing Algorithms

The vast majority of literature in Arrow-Debreu or Fisher markets focuses either on the existence of equilibriums or on centralized algorithms for their computation. Albeit significant, such solutions cannot be applied in practice, especially in the sharing economy applications we consider here. Instead, it is of utmost importance to understand if (and how) the agents in such economies can make allocation decisions that will gradually drive the system to its equilibrium, without the need for a central coordinator or full knowledge about the market state. In [9] the authors studied exactly this problem and proposed a new distributed algorithm, called the Proportional Response (PR) dynamics, as a protocol for trading bandwidth on a peer-to-peer file sharing network. The PR dynamics involve a sequence of bids by buyers and exchange ratios by sellers that respond to each other. The seller exchange ratios are simply set to be the sum of all the bids they receive. The buyers set their bids proportional to the utility they would obtain with the bids and the exchange ratios in the previous round. Subsequently, [40] studied the application of these dynamics for trading in a Fisher market, while [41] showed that the proportional bidding and allocation algorithm introduced in [9] is essentially a gradient descent algorithm that solves the Fisher market problem in a decentralized fashion.

This proportional allocation algorithm is substantially different from the tatonnement process. In the latter, the exchange ratio of each good is gradually adjusted according to the excess demand in the previous time step, and the agents update their requests based on the new ratios. The PR dynamics on the other hand do not require an exchange ratio mechanism as the requests are based on the user's utility. Consequently, it does not need a central controller to aggregate the demands and offers, nor it requires solving an optimization problem. However, all the above approaches refer to a static model, where the amounts of commodities and budgets are constant and known in advance. In this work we consider a more challenging and realistic system where the resource availability varies randomly, and we prove the convergence of a distributed algorithm that fully allocates the generated commodities to the requester offering the highest sharing ratio. This decision rule is intuitive and in line with the expected market behavior of agents, and compatible with our recent experimental findings in [22].

5.3 Sharing in Communication & Computing Systems

The problem of cooperation in wireless networks is of paramount importance and has been studied in different cases, such as for ad hoc networks [45] or WiFi sharing models [10], while recently gaining increasing interest again [11]. Indeed, today there are many pertinent market solutions offered either as a product, e.g., routers that are tailored for cooperation [7], or mobile applications that allow sharing of content among devices. Moreover, such mechanisms have been extensively studied in file sharing peer-to-peer (P2P) overlay systems. In this case, each device that participates in the overlay is an economic agent in the sense that it provides some commodity (the files it possesses) and requests some other goods (the files other devices share). While our model is similar to previous works, e.g., see [8] and references therein, our analysis provides novel insights for the structure and properties of the resulting equilibriums and focuses, for the first time, in the dynamics of such interactions. Finally, in more grassroots efforts, such cooperative infrastructures offer low-cost Internet access to under-served or low-income communities around the globe [20]. Unlike previous works, our model does not presume any kind of logistics infrastructure, e.g., for transaction or reputation systems. Instead, we proved that a simple and intuitive best response algorithm, with no information about the graph and resource endowments, converges to a fair and robust connectivity or content sharing equilibrium.

5.4 Sharing Economy

An important thrust in sharing economy consists of works analyzing participation motives. It is found that users join such platforms often due to sustainability concerns [44], [49] and their decisions are facilitated when mediators resolve the logistic and security issues [47]. Our focus in this paper is on decentralized sharing economy models where users hold responsibility of and directly exchange their resources [5]. And we also study the relation of the network graph and the sharing equilibriums. This is particularly important for the additional reason that there is recent empirical evidence that motifs in cooperative networks impact both their efficiency and, also, each node's welfare; see [48].

In sharing systems the behavioral reciprocity mechanisms are crucial [50], [55], [58]; and the collaboration is affected by the graph. This was first identified by sociologists [42], [52], [57] who studied how an actor's network position affects its benefits when involved in bargains. The main hypothesis was that actors behave strategically and decrease their offers when they are succeeding in ensuring collaborations; and increase them otherwise. Using this model, [46] studied analytically under what network conditions we obtain bargaining equilibriums. Our model is fundamentally different as it assumes a competitive behavioral model, namely users respond to prices and do not anticipate their neighbors actions (typical difference between price equilibriums and non-cooperative equilibriums), which is also supported by experiments in exchange theory [56]. Importantly, in a series of recent behavioral experiments targeting exactly this model, namely in the context of a Wi-Fi sharing scenario, we found that humans exhibit behavior that is very close to our theoretical model [22], and that their network position indeed affects their performance.

6 Discussion and Conclusions

6.1 Model Extensions

Our model can be substantially extended, covering an even larger set of scenarios. First, the presented policy is distributed since each node needs to be informed only about the sharing ratios of its outgoing neighbors. As it stands now however, each node has to know its own average resource generation rate D_i. Interestingly, this requirement can also be removed by replacing $D_i(t)$ with

$$\bar{D(t)} = \frac{\sum_{\tau=1}^{t} D_i(\tau)}{t}.$$

The arguments for proving this claim are mainly technical, albeit lengthy; hence we refer the interested reader to [25] where a different system with similar dynamics is analyzed. In practice, this means that the proposed algorithm can drive the sharing economy network to the desirable operation point with minimum local interactions among nodes, and with no information about the actual resource availability (statistics of resource generation).

Another issue with Algorithm 1 is that if the statistics of node endowments change, given that the decisions are based on time averages, the system adaptation to new parameters could be slow. This issue however can be avoided by replacing time averages with their discounted versions, e.g., by replacing $\bar{D_i}(t)$ with $\hat{D}_i(t) = (1-\alpha)D_i(t) + \alpha\hat{D}_i(t-1), t \geq 2,\ \hat{D}_i(1) = D_i(1),\ 0 < \alpha < 1$. It can be shown that by selecting α close to 1, the system performance is close to optimal, while ensuring satisfactory speed of adaptation to statistical changes of parameters. We refer the reader to [25] for more details on this.

Finally, we have assumed that the resources of the different users are of equal importance for their neighbors. This scenario captures many practical systems where indeed the exchanged resources are equivalent in value (or, cannot be differentiated for practical reasons, e.g., consider a time-bank). However, our analysis and results hold for the more general case where each user's resource has a different priority or benefit for its neighbors. Let vector $\boldsymbol{a} = (a_1, a_2, \ldots, a_N)$ denote the (normalized) importance of the resources of the N nodes. Then, when i allocates $D_{ij}(t)$ units of resource to node j, it essentially assigns to it utility equal to $U_{ij}(t) = a_i D_{ij}(t)$. It holds also:

$$\sum_{\tau=0}^{t} \sum_{j \in \mathcal{N}_i} U_{ij}(\tau) = \sum_{\tau=0}^{t} \sum_{j \in \mathcal{N}_i} a_i D_{ij}(\tau) =$$
$$= a_i \sum_{\tau=0}^{t} \sum_{j \in \mathcal{N}_i} D_{ij}(\tau) \leq a_i \sum_{\tau=0}^{t} D_i(\tau) = \sum_{\tau=0}^{t} U_i(\tau),$$

and hence the average long-term utility that i can give to the network is $U_i = a_i D_i$. On the other hand, the utility each node $i \in \mathcal{N}$ obtains at time t is:

$$R_i(t) = \sum_{j \in \mathcal{N}_i} a_j D_{ji}(t) = \sum_{j \in \mathcal{N}_i} U_{ji}(t).$$

Therefore, our model can be extended for this more rich scenario by replacing D_i with $a_i D_i$ for every node $i \in \mathcal{N}$.

6.2 Final Remarks

We introduced and analyzed a novel model of decentralized sharing economy networks where agents embedded in a graph share directly their resources over time. This basic model captures an increasing number of business cases where end-users exchange resources or services in a direct and decentralized fashion. Such solutions are fueled today by the ideas about collaborative consumption, or sharing economy, and have the potential to boost the global economy in many ways, transforming the way humans trade and collaborate. End users and communities are expected to play a key role in this new era, and hence the problem of analyzing the efficiency of the resulting equilibriums is crucial [5]. Our findings suggest that there is a simple class of dynamic exchange policies that can lead to an equilibrium point which is fair, in a max-min fashion, and stable with respect to selfish strategies of single or groups of colluding agents. Moreover, the equilibrium allocations have an interesting structure that underline the relation of sharing equilibriums with the network graph topology.

APPENDIX

Proof of Lemma 1

According to the definitions and eq. (1):

$$\sum_{i\in\mathcal{S}} \bar{R}_i^{\pi}(t) = \frac{\sum_{i\in\mathcal{S}}\sum_{\tau=1}^{t}\sum_{j\in\mathcal{N}_i} R_{ji}(\tau)}{t} =$$
$$\frac{\sum_{j\in\mathcal{N}_\mathcal{S}}\sum_{\tau=1}^{t}\sum_{i\in\mathcal{N}_j} R_{ji}(\tau)}{t} \leq \frac{\sum_{j\in\mathcal{N}_\mathcal{S}}\sum_{\tau=1}^{t} D_j(\tau)}{t}.$$

Taking limits and using $\lim_{t\to\infty}\left(\sum_{\tau=1}^{t} D_j(\tau)\right)/t = D_i$, (4) follows. And from (4),

$$\sum_{i\in\mathcal{S}} r_i^{\pi} = \sum_{i\in\mathcal{S}} \liminf_{t\to\infty} \bar{R}_i^{\pi}(t) \leq \liminf_{t\to\infty} \sum_{i\in\mathcal{S}} \bar{R}_i^{\pi}(t) \leq f(\mathcal{S}). \blacksquare$$

Proof of Lemma 2

By a theorem of Lovasz [51], it suffices to show that for any $k\in\mathcal{N}$, and $\mathcal{T}\subseteq\mathcal{S}\subseteq\mathcal{N}-\{k\}$ it holds,

$$f(\mathcal{S}\cup\{k\}) - f(\mathcal{S}) \leq f(\mathcal{T}\cup\{k\}) - f(\mathcal{T}).$$

Indeed,

$$f(\mathcal{T}\cup\{k\}) - f(\mathcal{T}) = \sum_{i\in\mathcal{N}_{\mathcal{T}\cup\{k\}}} D_i - \sum_{i\in\mathcal{N}_\mathcal{T}} D_i =$$
$$\sum_{i\in\mathcal{N}_k-\mathcal{N}_\mathcal{T}} D_i \geq \sum_{i\in\mathcal{N}_k-\mathcal{N}_\mathcal{S}} D_i \ \text{ since } \mathcal{T}\subseteq\mathcal{S}$$
$$= f(\mathcal{S}\cup\{k\}) - f(\mathcal{S}). \blacksquare$$

Proof of Lemma 3

Observe first that $\mathcal{R}_0$ is convex since given two policies $\pi_1, \pi_2 \in \Pi_0$, one can design a policy $\pi_3\in\Pi_0$ with $\boldsymbol{r}^{\pi_3} = p\boldsymbol{r}^{\pi_1} + (1-p)\,\boldsymbol{r}^{\pi_2}$, where $p\in[0,1]$, as follows:

$$D_{ij}^{\pi_3}(t) = pD_{ij}^{\pi_1}(t) + (1-p)\,D_{ij}^{\pi_2}(t).$$

Since $\mathcal{R}_0$ is convex and $\mathcal{R}_0 \subseteq \mathcal{A}_0$ to show that in fact $\mathcal{R}_0 = \mathcal{A}_0$, it suffices to show that all extreme points of $\mathcal{A}_0$ belong to $\mathcal{R}_0$. Since $\mathcal{A}_0$ is a base of a polymatroid, its extreme points are defined as follows. Let $\sigma(i)$ be any permutation of node indices. Define also $\mathcal{S}_\sigma(i) = \{\sigma(1),\ldots,\sigma(i)\}$. Then an extreme point of $\mathcal{A}_0$ is the following,

$$r_{\sigma(1)} = f(\mathcal{S}_\sigma(1)), \tag{12}$$
$$r_{\sigma(i)} = f(\mathcal{S}_\sigma(i)) - f(\mathcal{S}_\sigma(i-1)),\ 2\leq i\leq N, \tag{13}$$

and in fact all extreme points of $\mathcal{A}_0$ are of the form (12), (13). Consider the policy π^σ that operates as follows:

- All nodes in $\mathcal{N}_{\sigma(1)}$ give always their resource to $\sigma(1)$.
- All nodes in $\mathcal{N}_{\sigma(k)} - \cup_{l=1}^{k-1}\mathcal{N}_{\sigma(l)}$ give always their resource to $\sigma(k)$, $2\leq k\leq N$.

It is clear from the definitions that under policy π^σ the long-term resources allocated to nodes are given by (12), (13). $\blacksquare$

Proof of Lemma 5

Since $\boldsymbol{r}\in\mathcal{A}_0$, we have

$$\sum_{i\in\mathcal{N}} r_i = f(\mathcal{N}) = \sum_{i\in\mathcal{N}} D_i. \tag{14}$$

(i) If $K=1$, then $r_i = v_1 D_i$ for all $i\in\mathcal{N}$, hence $v_1 = 1$.
(ii) Let $K>1$. If $v_1\geq 1$, then since $v_k > v_1$, $k\geq 2$, we have:

$$\sum_{i\in\mathcal{N}} r_i = \sum_{k=1}^{K} v_k \sum_{i\in\mathcal{L}_k} D_i > \sum_{i\in\mathcal{N}} D_i, \tag{15}$$

which contradicts (14). Similarly it is shown that $v_K > 1$. $\blacksquare$

Proof of Theorem 1

Here we show that policy π^* that operates according to Algorithm 1 is lexicographically optimal. The rest of the assertions of the theorem follow from the discussion in Section 3.4. The proof is based on stochastic approximation techniques. In particular, we will make use of the following stochastic approximation theorem due to Robbins and Siegmund and follow the approach in [25].

Theorem 6. *On a probability space $(\Omega,\mathcal{F},P)$ equipped with a sequence of σ-fields $\mathcal{F}_0\subset\cdots\subset\mathcal{F}_t\subset\mathcal{F}$ let $z_t,\ \xi_t,\ \zeta_t$ be non-negative and $\mathcal{F}_t$-measurable random variables such that*

$$E[z_{t+1}|\mathcal{F}_t] \leq z_t - \zeta_t + \xi_t,\ t=1,\cdots,$$

where

$$\sum_{t=1}^{\infty}\xi_t < \infty\ a.s. \tag{16}$$

Then, $\lim_{t\to\infty} z_t$ exists, is finite and $\sum_{t=1}^{\infty}\zeta_t<\infty$ a.s.

We will need the following lemma.

Lemma 6. *Let $J_i(\boldsymbol{r}) = E\left[R_i(t+1)\,|\bar{\boldsymbol{R}}(t) = \boldsymbol{r}\right]$, $\boldsymbol{r}\geq\boldsymbol{0}$. It holds:*

$$\sum_{i\in\cup_{k=1}^{n}\mathcal{L}_k(\boldsymbol{r})} J_i(r) = f\left(\cup_{k=1}^{n}\mathcal{L}_k(\boldsymbol{r})\right),\ n=1,\cdots,K(\boldsymbol{r}). \tag{17}$$

For all $\epsilon>0$ it is:

$$\inf_{\|\boldsymbol{r}-\boldsymbol{r}^*\|\geq\epsilon} \sum_{i=1}^{N}\frac{1}{D_i}(r_i^*-r_i)(J_i(\boldsymbol{r})-r_i) > 0. \tag{18}$$

Proof. According to Algorithm 1, the nodes in $\mathcal{L}_1(\boldsymbol{r})$ receive all the resources of their neighbors at time $t+1$.

Similarly, the nodes in $\cup_{k=1}^{n}\mathcal{L}_n(\boldsymbol{r})$ receive all the resources generated by their neighbors at time $t+1$, hence:

$$\sum_{i\in\cup_{n=1}^{k}\mathcal{L}_n(\boldsymbol{r})} R_i(t+1) = \sum_{i\in\cup_{n=1}^{k}\mathcal{L}_n(\boldsymbol{r})} D_i(t+1).$$

Taking conditional expectations we obtain:

$$\sum_{i\in\cup_{k=1}^{n}\mathcal{L}_1\boldsymbol{r}} J_i(r) = \sum_{i\in\cup_{k=1}^{n}\mathcal{L}_n(\boldsymbol{r})} D_i = f\left(\cup_{k=1}^{n}\mathcal{L}_k(\boldsymbol{r})\right).$$

To show (18), setting $C=\min_{i=1,\dots,N}\{1/D_i\}$ we have for $\|\boldsymbol{r}-\boldsymbol{r}^*\|\geq\epsilon$,

$$\sum_{i=1}^{N}\frac{1}{D_i}(r_i^*-r_i)(J_i(\boldsymbol{r})-r_i)=\sum_{i=1}^{N}\frac{1}{D_i}(r_i^*-r_i)(J_i(\boldsymbol{r})-r_i^*)$$
$$+\sum_{i=1}^{N}\frac{1}{D_i}(r_i^*-r_i)^2\geq$$
$$\sum_{i=1}^{N}\frac{1}{D_i}r_i^*(J_i(\boldsymbol{r})-r_i^*)-\sum_{i=1}^{N}\frac{1}{D_i}r_i(J_i(\boldsymbol{r})-r_i^*)+\epsilon^2C.$$

Hence to prove (18) it suffices to show that

$$\sum_{i=1}^{N}\frac{1}{D_i}r_i(J_i(\boldsymbol{r})-r_i^*)\leq 0, \tag{19}$$

and

$$\sum_{i=1}^{N}\frac{1}{D_i}r_i^*(J_i(\boldsymbol{r})-r_i^*)\geq 0. \tag{20}$$

Next we describe the structure of the lexicographically optimal vector in $\mathcal{A}$. To show (19) write,

$$\sum_{i=1}^{N}\frac{1}{D_i}r_i\big(J_i(\boldsymbol{r})-r_i^*\big)=\sum_{k=1}^{K(\boldsymbol{r})}v_k(\boldsymbol{r})\sum_{i\in\mathcal{L}_k(\boldsymbol{r})}(J_i(\boldsymbol{r})-r_i^*)$$
$$=\sum_{k=1}^{K(\boldsymbol{r})-1}\left(\sum_{n=k}^{K(\boldsymbol{r})-1}(v_n(\boldsymbol{r})-v_{n+1}(\boldsymbol{r}))+v_{K(\boldsymbol{r})}(r)\right)$$
$$\cdot\left(\sum_{i\in\mathcal{L}_k(\boldsymbol{r})}J_i(\boldsymbol{r})-\sum_{i\in\mathcal{L}_k(\boldsymbol{r})}r_i^*\right)$$
$$+v_{K(\boldsymbol{r})}(\boldsymbol{r})\left(\sum_{i\in\mathcal{L}_{K(\boldsymbol{r})}(\boldsymbol{r})}J_i(\boldsymbol{r})-\sum_{i\in\mathcal{L}_{K(\boldsymbol{r})}(\boldsymbol{r})}r_i^*\right)$$
$$=\sum_{k=1}^{K(\boldsymbol{r})-1}\left(\sum_{n=k}^{K(\boldsymbol{r})-1}v_n(\boldsymbol{r})-v_{n+1}(\boldsymbol{r})\right)\left(\sum_{i\in\mathcal{L}_k(\boldsymbol{r})}J_i(\boldsymbol{r})-r_i^*\right)$$
$$+v_{K(\boldsymbol{r})}(\boldsymbol{r})\left(\sum_{i\in\mathcal{N}}J_i(\boldsymbol{r})-\sum_{i\in\mathcal{N}}r_i^*\right)$$
$$=\sum_{n=1}^{K(\boldsymbol{r})-1}(v_n(\boldsymbol{r})-v_{n+1}(\boldsymbol{r}))\left(\sum_{i\in\cup_{k=1}^{n}\mathcal{L}_k(\boldsymbol{r})}J_i(\boldsymbol{r})-r_i^*\right)$$
$$+v_{K(\boldsymbol{r})}(\boldsymbol{r})\left(\sum_{i\in\mathcal{N}}J_i(\boldsymbol{r})-\sum_{i\in\mathcal{N}}r_i^*\right)$$
$$=\sum_{n=1}^{K(\boldsymbol{r})-1}(v_n(\boldsymbol{r})-v_{n+1}(\boldsymbol{r}))\left(f\left(\cup_{k=1}^{n}\mathcal{L}_k(\boldsymbol{r})\right)-\sum_{i\in\cup_{k=1}^{n}\mathcal{L}_k(\boldsymbol{r})}r_i^*\right)$$
$$+v_{K(\boldsymbol{r})}(\boldsymbol{r})\left(f(\mathcal{N})-\sum_{i\in\mathcal{N}}r_i^*\right)\text{ by (17)}$$
$$\leq 0,\text{ since }\boldsymbol{r}^*\in\mathcal{R}_0\text{ and }v_n(\boldsymbol{r})\leq v_{n+1}(\boldsymbol{r}),\ n=1,\dots K(\boldsymbol{r})-1.$$

To show (20) we repeat the same procedure but summing over indices $l_k^{\boldsymbol{r}^*}, k=1,\dots,K^{\boldsymbol{r}^*}$. ■

We now proceed to show that π^* is lex-optimal, i.e.:

$$\lim_{t\to\infty}\bar{R}_i(t)=\boldsymbol{r}^*.$$

Write:

$$\bar{R}_i(t+1)=\frac{\sum_{\tau=1}^{t+1}R_i(t)}{t+1}=\frac{t}{t+1}\frac{\sum_{\tau=1}^{t}R_i(t)}{t}+\frac{R_i(t+1)}{t+1}$$
$$=\frac{t}{t+1}\bar{R}_i(t)+\frac{R_i(t+1)}{t+1}$$
$$=\bar{R}_i(t)+\frac{1}{t+1}\left(R_i(t+1)-\bar{R}_i(t)\right), \tag{21}$$

and consider the Lyapunov function,

$$V(\boldsymbol{x})=\frac{1}{2}\sum_{i=1}^{N}\frac{1}{D_i}(x_i-r_i^*)^2.$$

From (21) we obtain, $V\left(\bar{\boldsymbol{R}}(t+1)\right)=$

$$=\frac{1}{2}\sum_{i\in\mathcal{N}}\frac{1}{D_i}\left(\bar{R}_i(t)+\frac{1}{t+1}\left(R_i(t+1)-\bar{R}_i(t)\right)-r_i^*\right)^2$$
$$=V\left(\bar{\boldsymbol{R}}(t)\right)-\frac{1}{t+1}\sum_{i\in\mathcal{N}}\frac{1}{D_i}\left(R_i(t+1)-\bar{R}_i(t)\right)\left(r_i^*-\bar{R}_i(t)\right)$$
$$+\frac{1}{2(t+1)^2}\sum_{i\in\mathcal{N}}\frac{1}{D_i}\left(R_i(t+1)-\bar{R}_i(t)\right)^2.$$

Applying now Theorem 6 with $z_t=V\left(\bar{\boldsymbol{R}}(t)\right)$, we get $\zeta_t=$

$$\frac{1}{t+1}E\left[\sum_{i\in\mathcal{N}}\frac{1}{D_i}\left(R_i(t+1)-\bar{R}_i(t)\right)\left(r_i^*-\bar{R}_i(t)\right)|\bar{R}_i(t)\right]$$
$$=\frac{1}{t+1}\sum_{i\in\mathcal{N}}\frac{1}{D_i}\left(J_i\left(\bar{\boldsymbol{R}}(t)\right)-\bar{R}_i(t)\right)\left(r_i^*-\bar{R}_i(t)\right),$$

and

$$\xi_t=\frac{1}{2(t+1)^2}E\left[\sum_{i\in\mathcal{N}}\frac{1}{D_i}\left(R_i(t+1)-\bar{R}_i(t)\right)^2|\bar{R}_i(t)\right].$$

Clearly, $z_t\geq 0$ and $\xi_t\geq 0$. Also, according to (18), $\zeta_t\geq 0$. Hence the non-negativity of the variables in the theorem holds. Notice that

$$\bar{R}_i(t)=\frac{\sum_{\tau=1}^{t}R_i(\tau)}{t}\leq\sum_{j\in\mathcal{N}}\frac{\sum_{\tau=1}^{t}D_j(\tau)}{t}\leq NB,\ \text{thus:}$$

$$\xi_t \leq \frac{1}{(t+1)^2} E\left[\sum_{i\in\mathcal{N}} \frac{1}{D_i}\left(R_i^2(t+1)+\bar{R}_i^2(t)\right)|\bar{R}_i(t)\right] =$$
$$\frac{1}{(t+1)^2} E\left[\sum_{i\in\mathcal{N}} \frac{1}{D_i} R_i^2(t+1)\,|\bar{R}_i(t)\right] + \frac{1}{(t+1)^2}\sum_{i\in\mathcal{N}} \frac{1}{D_i}\bar{R}_i^2(t)$$
$$\leq \frac{\hat{B}}{(t+1)^2},$$

where $\hat{B} = N^2 B^2 \sum_{i\in\mathcal{N}} 1/D_i$. It follows that $\lim_{t\to\infty}\sum_{t=1}^{\infty}\xi_t < \infty$, i.e. (16) holds. Hence, according to Theorem 6 we have that $\lim_{t\to\infty} V\left(\bar{\boldsymbol{R}}(t)\right)$ exists almost surely and that

$$\sum_{t=1}^{\infty}\frac{1}{t+1}\sum_{i\in\mathcal{N}}\frac{1}{D_i}\left(J_i\left(\bar{\boldsymbol{R}}(t)\right)-\bar{R}_i(t)\right)\left(r_i^*-\bar{R}_i(t)\right) < \infty \; a.s. \tag{22}$$

We will show next that $\lim_{t\to\infty} V\left(\bar{\boldsymbol{R}}(t)\right) = 0$ which implies that $\lim_{t\to\infty}\bar{\boldsymbol{R}}(t) = \boldsymbol{r}^*$ i.e., policy π^* is lex-optimal.

Assume that $\lim_{t\to\infty} V\left(\bar{\boldsymbol{R}}(t)\right) = \alpha > 0$. Since $\left\|\bar{\boldsymbol{R}}(t) - r^*\right\| \geq 2CV\left(\bar{\boldsymbol{R}}(t)\right)$, where $C = \min_{i\in\mathcal{N}} D_i$, we conclude that

$$\liminf_{t\to\infty}\left\|\bar{\boldsymbol{R}}(t) - r^*\right\| \geq 2C\alpha > 0,$$

which by (18) implies that

$$\liminf_{t\to\infty}\sum_{i\in\mathcal{N}}\frac{1}{D_i}\left(J_i\left(\bar{\boldsymbol{R}}(t)\right)-\bar{R}_i(t)\right)\left(r_i^*-\bar{R}_i(t)\right) > 0, \text{ hence:}$$

$$\sum_{t=1}^{\infty}\frac{1}{t+1}\sum_{i\in\mathcal{N}}\frac{1}{D_i}\left(J_i\left(\bar{\boldsymbol{R}}(t)\right)-\bar{R}_i(t)\right)\left(r_i^*-\bar{R}_i(t)\right) = \infty$$

which contradicts (22). ■

Proof of Theorem 3

Let $K=1$ and $\boldsymbol{r}$ be lex-optimal, hence $\boldsymbol{r}\in\mathcal{A}_0$. Then according to Lemma 5, $v_1 = r_i/D_i = 1,\ i\in\mathcal{N}$. If on the other hand $v_1 = 1$ then:

$$\sum_{i\in\mathcal{N}} r_i = \sum_{i\in\mathcal{N}} D_i = f(\mathcal{N}),$$

and according to Theorem 2, $\boldsymbol{r}$ is lexicographically optimal. Next we consider the case $K\geq 2$.

(i) Assume that $\boldsymbol{r}$ is lexicographically optimal and hence

$$\sum_{i\in\mathcal{L}_1} r_i = f(\mathcal{L}_1) = \sum_{i\in\mathcal{N}_{\mathcal{L}_1}} D_i. \tag{23}$$

We first show that $\mathcal{L}_1$ is an independent set. Assume that there are two nodes in $\mathcal{L}_1$ that are connected and consider the maximal connected set $\mathcal{L}$ in $\mathcal{L}_1$ that contains these two nodes. Fix any allocation set that generates $\boldsymbol{r}$. Since $\mathcal{L}$ is maximal, no node in $\mathcal{L}$ is connected to a node in $\mathcal{L}_1$; then, (23) implies that (under the allocation that generates $\boldsymbol{r}$) all nodes in $\mathcal{L}$ give their endowment to nodes in $\mathcal{L}$ and hence,

$$\sum_{i\in\mathcal{L}} r_i \geq \sum_{i\in\mathcal{L}} D_i, \;\Rightarrow\; v_1\sum_{i\in\mathcal{L}} D_i \geq \sum_{i\in\mathcal{L}} D_i$$

or $v_1 \geq 1$, which contradicts Lemma 5.

Consider now the smallest level m such that nodes in $\mathcal{L}_1$ are connected to some nodes in $\mathcal{L}_m$. Let $\mathcal{S}_1$ be the set of nodes in $\mathcal{L}_1$ which are connected to some nodes in $\mathcal{L}_m$. Let $\mathcal{S}_m$ be the set of nodes in $\mathcal{L}_m$ that are connected to some nodes in $\mathcal{S}_1$. Then, since according to Theorem 2 the nodes in $\mathcal{S}_1$ receive all the endowments of nodes in $\mathcal{S}_m$, we have

$$\sum_{i\in\mathcal{S}_1} r_i \geq \sum_{m\in\mathcal{S}_m} D_m, \text{ or } v_1\sum_{i\in\mathcal{S}_1} D_i \geq \sum_{m\in\mathcal{S}_m} D_m\,. \tag{24}$$

Also, since nodes in $\mathcal{S}_1$ are not connected, the nodes in $\mathcal{S}_m$ receive all the endowments of nodes in $\mathcal{S}_1$, hence

$$\sum_{i\in\mathcal{S}_m} r_i \geq \sum_{i\in\mathcal{S}_1} D_i, \text{ or } v_m\sum_{i\in\mathcal{S}_m} D_i \geq \sum_{i\in\mathcal{S}_1} D_i\,. \tag{25}$$

From (24) and (25) and the fact that $\sum_{i\in\mathcal{S}_m} D_i > 0$, $\sum_{i\in\mathcal{S}_1} D_i > 0$, we conclude that

$$v_1 v_K \geq v_1 v_m \geq 1. \tag{26}$$

Next consider the set $\mathcal{L}_K$. Notice first that if node i gives some of its endowment to nodes in $\mathcal{L}_K$, then $\mathcal{N}_i\in\mathcal{L}_K$. This is so, since if node i has a neighbor in a set $\mathcal{L}_n,\ n<K$ then according to Theorem 2 node i would give all its endowment to nodes with lower levels. It follows that no node in $\mathcal{L}_K$ gives endowment to $\mathcal{L}_K$. To see this, let $\mathcal{S}\neq\emptyset$ be the set of nodes in $\mathcal{L}_K$ that give their endowment to $\mathcal{L}_K$. Then the nodes in $\mathcal{S}$ receive endowments only from nodes in $\mathcal{S}$. This is so, because if node i gives endowment to $j\in\mathcal{S}$ then $j\in\mathcal{N}_i$, hence $j\in\mathcal{L}_K$ and then by definition of the set $\mathcal{S}$, $i\in\mathcal{S}$. Hence, it holds

$$\sum_{i\in\mathcal{S}} r_i \leq \sum_{i\in\mathcal{S}} D_i, \text{ or } v_k\sum_{i\in\mathcal{S}} D_i \leq \sum_{i\in\mathcal{S}} D_i\,. \tag{27}$$

i.e., $v_i \leq 1$ which contradicts Lemma 5.

Let now $n<K$ be the largest level such that $\mathcal{L}_n$ contains a node that gives endowment to $\mathcal{L}_K$. Let $\mathcal{S}_n$ be the nodes in $\mathcal{L}_n$ that give endowment to some node in $\mathcal{L}_K$. Note that the nodes in $\mathcal{S}_n$ are connected only to nodes in $\mathcal{S}_K$ since otherwise they (the nodes in $\mathcal{S}_n$) would give their endowment to nodes at lower levels. Let $\mathcal{S}_K$ be the nodes in $\mathcal{N}_{\mathcal{S}_n}\cap\mathcal{L}_K$ that give endowment to nodes in $\mathcal{S}_n$. Then, since the nodes in $\mathcal{S}_n$ are connected only to nodes in $\mathcal{S}_K$, it holds

$$\sum_{i\in\mathcal{S}_n} r_i \leq \sum_{i\in\mathcal{S}_K} D_i, \text{ or } v_n\sum_{i\in\mathcal{S}_n} D_i \leq \sum_{i\in\mathcal{S}_K} D_i. \tag{28}$$

Also, note that the nodes in $\mathcal{S}_K$ are not connected to nodes at lower levels than n since otherwise they would give their endowment to these nodes. Since we already showed that these nodes do not get any endowment from nodes in $\mathcal{L}_K$, it follows that

$$\sum_{i\in\mathcal{S}_K} r_i \leq \sum_{i\in\mathcal{S}_n} D_i, \text{ or } v_K\sum_{i\in\mathcal{S}_K} D_i \leq \sum_{i\in\mathcal{S}_n} D_i. \tag{29}$$

From (28), (29) we conclude

$$v_K v_1 \leq v_K v_n \leq 1. \tag{30}$$

Inequalities (26), (30) imply that $v_K v_1 = 1,\ m=K$, and $n=1$ and these imply statements of the theorem for $k=1$.

Next consider the graph $G_{\mathcal{Q}_2(\boldsymbol{r})} = \left(\mathcal{Q}_2(\boldsymbol{r}), \mathcal{E}_{\mathcal{Q}_2}(\boldsymbol{r})\right)$ and the vector $\boldsymbol{r}_2$ that has components those of vector $\boldsymbol{r}$ that are in $\mathcal{N}-(\mathcal{L}_1\cup\mathcal{L}_K)$. It can be easily seen that $\boldsymbol{r}_2$ is lexicographically optimal in $G_{\mathcal{Q}_2(\boldsymbol{r})}$ and therefore we can repeat the process to complete the proof using induction.

Assume now that conditions 1-5 of the theorem hold. Then it can be seen by induction on k that conditions (9),

(10) of Theorem 2 hold, and hence the vector $\boldsymbol{r}$ is lexicographically optimal. To see this, we describe the case $k=1$. Since by condition 1 $\mathcal{L}_1$ is an independent set, we conclude from conditions 2 and 4 that the nodes in $\mathcal{L}_1$ receive all endowments of their neighbors, hence (9) of Theorem 2 is satisfied. Also,

$$\sum_{i\in\mathcal{L}_K} r_i = v_K \sum_{i\in\mathcal{L}_K} D_i = \frac{1}{v_1}\sum_{i\in\mathcal{L}_1} r_i = \sum_{i\in\mathcal{L}_1} D_i,$$

where the second equality holds due to conditions 3 and 4. The last equality implies that nodes in $\mathcal{L}_K$ receive only the resources of nodes in $\mathcal{L}_1$ which are not connected to nodes of lower level than K, hence (10) is satisfied for $k=K$. ■

Proof of Theorem 4

Notice that $\rho_i^* = v_{I_i}^*$. Since $\boldsymbol{r}^* \in \mathcal{A}_0$, there is an allocation set $\left\{d_{ij}^* \geq 0,\ i\in\mathcal{N},\ j\in\mathcal{N}_i\right\}$, such that

$$\sum_{j\in\mathcal{N}_i} d_{ij}^* = D_i,\ i\in\mathcal{N}, \tag{31}$$

$$\sum_{j\in\mathcal{N}_i} d_{ji}^* = r_i,\ i\in\mathcal{N}. \tag{32}$$

Consider the following policy π^* that operates as follows. At any time t, node i allocates to node $j\in\mathcal{N}_i$ resource $D_{ij}(t) = \frac{d_{ij}^*}{D_i}$. This implies that under policy π^* (3) is satisfied, i.e., $\lim_{t\to\infty}\bar{D}(t) = D_i$. Moreover, it is easily shown that

$$\lim_{t\to\infty} \bar{R}_i^{\pi^*}(t) = r_i^* = D_i\rho_i^*.$$

Finally, from the structure of $\boldsymbol{r}^*$ in Theorem 3 it can be seen by induction on k that every node allocates the resources it generates to neighbors with the smallest sharing ratios. ■

Proof of Theorem 5

Assume that there is a set $\mathcal{S}\subset\mathcal{N}$ such that the nodes in this set exchange resources only between themselves and achieve rates $\hat{r}_i \geq r_i^*,\ i\in\mathcal{S}$ with $\hat{r}_j > r_j^*$ for at least one $j\in\mathcal{S}$. Let $\hat{\pi}$ be the policy that achieves rate vector $\hat{\boldsymbol{r}}$ and let $\hat{d_{ij}},\ i,j\in\mathcal{S}$ be an allocation vector that generates $\hat{\boldsymbol{r}}$, hence,

$$\sum_{j\in\mathcal{S}\cap\mathcal{N}_i} \hat{d}_{ji} = \hat{r}_i, i\in\mathcal{S}.$$

We then claim that

$$r_i^* < \hat{r}_i \Rightarrow \sqrt{\rho_i^*}D_i < \sum_{j\in\mathcal{S}\cap\mathcal{N}_i}\sqrt{\rho_j^*}\hat{d_{ji}}, \tag{33}$$

$$r_i^* \leq \hat{r}_i \Rightarrow \sqrt{\rho_i^*}D_i \leq \sum_{j\in\mathcal{S}\cap\mathcal{N}_i}\sqrt{\rho_j^*}\hat{d_{ji}}. \tag{34}$$

To see (33), assume that $\sqrt{\rho_i^*}D_i \geq \sum_{j\in\mathcal{S}\cap\mathcal{N}_i}\sqrt{\rho_j^*}\hat{d_{ji}}$. Since Theorems 3 and 4 imply that

$$\rho_j^* \geq \frac{1}{\rho_i^*},\ j\in\mathcal{N}_i,$$

we have,

$$\sqrt{\rho_i^*}D_i \geq \sum_{j\in\mathcal{S}\cap\mathcal{N}_i}\sqrt{\rho_j^*}\hat{d_{ji}} \geq \frac{1}{\sqrt{\rho_i^*}}\sum_{j\in\mathcal{S}\cap\mathcal{N}_i}\hat{d_{ji}} = \frac{1}{\sqrt{\rho_i^*}}\hat{r}_i. \tag{35}$$

Therefore, $r_i^* = \rho_i^* D_i \geq \hat{r}_i$, a contradiction. In a similar fashion (34) can be shown. Summing now (33), (34) over $i\in\mathcal{S}$ we have

$$\sum_{i\in\mathcal{S}}\sqrt{\rho_i^*}D_i < \sum_{i\in\mathcal{S}}\sum_{j\in\mathcal{S}\cap\mathcal{N}_i}\sqrt{\rho_j^*}\hat{d_{ji}} = \sum_{j\in\mathcal{S}}\sqrt{\rho_j^*}\sum_{i\in\mathcal{S}\cap\mathcal{N}_j}\hat{d_{ji}}$$
$$\leq \sum_{j\in\mathcal{S}}\sqrt{\rho_j^*}D_j, \tag{36}$$

a contradiction. ■

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